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The true times of the two transits at Chicago are T_2+E and t_2.
Hence. $D=T_2+E-t_2$...(2).
(1)+(2) gives $2D=T_1+T_2-t_1-t_2+E-e$...(3).
Similarly, when A and B have changed places, $2D=T'_1+T'_2-t'_1-t'_2-E+e$...(4).
$\frac{1}{4}$(3)+(4) gives $D=\frac{1}{4}(T_1+T_2+T_1+T_2-t_1-t_2-t'_1-t'_2)$. Substituting the values of the T's and t's, we have

$D=\frac{1}{4}$(3 hours, 19 minutes, 8 seconds) $=$ 49 minutes, 47 seconds, and
$L=15D=12^\circ\ 26'\ 45''$, the difference of longitude.

Also solved by V. M. Spunar.

GEOMETRY.

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335. **Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.**

Determine analytically, the point where three lines in a plane appear of equal length.

Solution by the PROPOSER.

In the most general solution we can take the rectangular axes of coordinates so that the axis of abscissas agrees, in direction, with one of the lines. Let this line be given by (0, 0); (a, 0); the second and third lines are then represented by the coordinates (b, c); (d, f), and (h, k); (m, n), respectively. Also let (x, y) be the point where they appear equal. The tangent of the angle formed by the lines through (x, y); (b, c) and (x, y); (d, f) is

$$\frac{\dfrac{c-y}{b-x}-\dfrac{f-y}{d-x}}{1+\dfrac{(c-y)\,(f-y)}{(b-x)\,(d-x)}}=\frac{cd-bf+(f-c)x+(b-d)y}{bd+cf-(b+d)x-(c+f)y+x^2+y^2}.$$

With similar expressions for the other lines.

$$\therefore\ \frac{ay}{ax-x^2-y^2}=\frac{cd-bf+(f-c)x+(b-d)y}{bd+cf-(b+d)x-(c+f)y+x^2+y^2}\ldots(1).$$

$$\frac{ay}{ax-x^2-y^2}=\frac{km-hn+(n-k)x+(h-m)y}{hm+kn-(h+m)x-(k+n)y+x^2+y^2}\ldots(2).$$

$$\begin{aligned}\therefore\ (f-c)x^3&+(cd-bf+ac-af+by-dy+ay)x^2\\&+(abf-acd-2aby+fy^2-cy^2)x+(abd+acf)y\\&+(cd-bf-ac-af)y^2+(a+b-d)y^3=0\ldots(3).\end{aligned}$$

$$(n-k)x^3+(km-hn+ak-an+hy-my+ay)x^2$$
$$+(ahn-akm-2ahy+ny^2-ky^2)x+(ahm+akn)y$$
$$+(km-hn-ak-an)y^2+(a+h-m)y^3=0\ldots(4).$$

(3) and (4) are the equations to determine x, y.

I. Let $c=f=k=n=0$. Then the three lines lie in the same straight line.

(3) becomes $(a+b-d)(x^2+y^2)=2abx-abd\ldots(5)$.

(4) becomes $(a+h-m)(x^2+y^2)=2ahx-ahm\ldots(6)$.

$$\therefore x=\frac{m(ah+bh+bd)-d(hm+ab+bd)}{2(ah-ab-dh+bm)}.$$

$$y=\pm\sqrt{\frac{2a(b-h)x+a(hm-bd)-(b-d-h+m)x^2}{b-d-h+m}}.$$

Let $a=b$, $h=d$. Then the three lines form one consecutive straight line.

$$\therefore x=\frac{3adm-d^2m-a^2d-ad^2}{2(ad-a^2-d^2+am)}$$

$$y=\pm\sqrt{\frac{2a(a-d)x+a(dm-ad)-(a-2d+m)x^2}{a-2d+m}}.$$

Let $d=2a$, $m=3a$. Then the lines are all of equal length.

$\therefore x=0/0$, and is indeterminate; $y=\infty$.

Hence there is no point at which three equal straight lines forming a continuous straight line will appear equal to each other.

II. $b=a$ and $c=0$ in (3) gives

$$(fx-2af+2ay-dy)(x^2+y^2)+a^2(fx-2xy+dy)=0\ldots(7).$$

$h=d$, $k=f$, $m=n=0$ in (4) gives

$$(fx-af-ay-dy)(x^2+y^2)+2ay(dx+fy)=0\ldots(8).$$

The three lines now are the sides of a triangle. The solution of these equations in general terms would be quite difficult, yet one can determine the values of x and y by the following method: Let u, v, w be the distances from (x, y) to $(0, 0)$; $(a, 0)$, (d, f), respectively. Then we have

$$x^2+y^2=u^2,\quad (a-x)^2+y^2=v^2,\quad (d-x)^2+(f-y)^2=w^2.$$

$$\therefore\ x=\frac{a^2+u^2-v^2}{2a},\quad y=\frac{(u^2-ad)(a-d)+dv^2+af^2-aw^2}{2af}.$$

Also, $u^2+uv+v^2=a^2$,

$u^2+uw+w^2=d^2+f^2$,

$v^2+vw+w^2=(a-d)^2+f^2$.

$$\therefore\ u=\frac{ad\pm\frac{1}{3}af\sqrt{3}}{\sqrt{(a^2+d^2+f^2-ad\pm af\sqrt{3})}},$$

$$v=\frac{a^2-ad\pm\frac{1}{3}af\sqrt{3}}{\sqrt{(a^2+d^2+f^2-ad\pm af\sqrt{3})}},$$

The plus sign is applicable to our solution.

$$w=\frac{d^2+f^2-ad\pm\frac{1}{3}af\sqrt{3}}{\sqrt{(a^2+d^2+f^2-ad\pm af\sqrt{3})}},$$

Let $d=\frac{1}{2}a$, and $f=\frac{1}{2}a\sqrt{3}$. Hence, $u=\frac{1}{3}a\sqrt{3}=v=w$. Thus the values of x and y are determined, and equations (7) and (8) are solved.

Hence $x=\dfrac{3ad^2+3af^2+3a^2d+a^2f\sqrt{3}+4adf\sqrt{3}}{6(a^2+d^2+f^2-ad+af\sqrt{3})}=\frac{1}{2}a$, when $d=\frac{1}{2}a$ and $f=\frac{1}{2}a\sqrt{3}$.

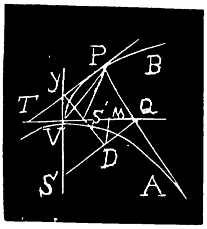
$$y=\frac{3a^2d\sqrt{3}-3ad^2\sqrt{3}+af^2\sqrt{3}+3a^2f}{6(a^2+d^2+f^2-ad+af\sqrt{3})}=\tfrac{1}{6}a\sqrt{3},\ \text{when } d=\tfrac{1}{2}a,\ f=\tfrac{1}{2}a\sqrt{3}.$$

Also solved by C. N. Schmall.

323. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.

S, *S′* are the foci of two co-vertical parabolas *A* and *B*, the axes of which are at right angles. Draw the circle *K* on *SS′* as diameter. K is cut in *D* and *E* by a straight line parallel to the axis of *A* such that *S′* lies midway between it and that axis. Show that the lines *S′D*, *S′E* are parallel to the two tangents to *A* which are normals to *B*.

II. Solution by (1) PROPOSER, and (2) R. F. DAVIS, M. A.



I°. The axis of *B* is the tangent at the common vertex *V* to *A*. Hence if *PQ* be normal at *P* to *B*, meeting the axis of *B* in *Q*, and if *PQ* is at the same time a tangent to *A*, it necessarily follows that *SQ* is perpendicular to *PQ* and therefore parallel to *TP* (the tangent at *P* to *B*). Or,

II°. *S′T*=*S′P*=*S′Q* (letters as above). Draw *YS′D* perpendicular to the parallels *TP*, *SQ*, and *DM* perpendicular to the axis of *B* (*Y*, of course, being on the tangent at *V* to *B*). Then *S′M*=*VS′*, and *D* is on the circle, *SS′* diameter, since ∠*SDS′*=90°.